# Incumbency in Imperfectly Discriminating Contests*

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**Abstract**

This paper considers a model in which contestants compete in two sequential imperfectly discriminating contests where the prize in each contest has a common but uncertain value, and the value of the prize in the first contest is positively related to that in the second. The contestant who obtains the prize in the first contest (the incumbent) privately observes its value, so that information in the second contest is asymmetric. Relative to the case where the prizes are independent random variables (so that the incumbent's private information does not provide a useful estimate of the value of the prize in the second contest), the incumbent is strictly better off, the other contestants (the challengers) are strictly worse off, and aggregate effort expenditures in the second contest strictly decrease. Further, aggregate effort expenditures in the first contest increases such that total effort expenditures over the two contests increase, relative to the case of independent prizes. Counterintuitively, the incumbent's ex ante probability of winning is strictly less than that of a challenger, despite expending (weakly) more effort than a challenger in expectation. In the second (terminal) contest, expected effort expenditure of an individual contestant is decreasing in the number of contestants, the expected utility of a contestant is decreasing in the number of contestants, and the aggregate expected effort expenditure is increasing in the number of contestants. Alternative methods of modeling an incumbency advantage are also considered.

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# 1 Introduction

Consider a group of workers who compete for a job with a particular firm. One of these workers prevails and begins working for the firm. Suppose that at some later date the firm seeks to fill a job opening which would be a promotion for the worker who got the job in the earlier competition. The group of workers is now in a position to compete for a second time. However, the second competition may be significantly different than the first.

In particular, it is natural to think that the worker who obtained the job in the first round will have more information than the other workers regarding the value of the second job. That is, she observes the intangible benefits of working for the firm, such as the corporate culture and how employees are treated. Consequently, the second competition is different than the first. Further, such asymmetric information in the second competition may affect the incentives in the first competition by increasing the value of winning.

This paper considers a model in which contestants compete in two sequential contests where the prize in each contest has a common but uncertain value, and the value of the prize in the first contest is positively related to that in the second contest. The contestant who obtains the prize in the first contest (the incumbent) privately observes its value, which provides a noisy estimate of the value in the second contest, thereby introducing asymmetric information. The contestants who do not obtain the prize in the first contest (the challengers) do not hold any private information in the second contest. Since contestants do not interact after the second contest, this framework allows me to examine the effect of information asymmetry on behavior in a one-shot game, as well as the effect on behavior when information asymmetry arises due to an incumbency advantage.

I utilize the well known model of imperfectly discriminating contests introduced in Tullock (1967). The associated literature is vast. Such a contest is a game in which economic agents expend unrecoverable effort in order to increase the probability of winning a prize. The contestant with the highest effort level does not win with certainty, but has the highest probability of winning.

Interestingly, I find that in the second contest, ex ante, the incumbent will expend weakly more effort than a challenger, but wins with a strictly lower probability. The intuition behind this result is that the incumbent expends little or no effort when she believes the value of the prize is low. As a result, the incumbent obtains the prize with low probability when its value is low. However, when the incumbent believes the value of the prize is high, she expends more effort than the challengers such that in expectation, the incumbent expends weakly more effort than a challenger. The incumbent's low effort expenditures when she believes the value of the prize is low dominates the higher effort expenditures when she believes the value of the prize is high, such that, ex ante, the incumbent's probability of obtaining the prize is strictly lower than that of a challenger.

I also find that, relative to the case where the value of the prizes in the two contests are independent (rendering the incumbent's private information strategically irrelevant), aggregate effort expenditures fall in the second contest, but increase in the first contest such that total effort expenditures summed over the two contests weakly increases.[1] This implies that, ex ante, contestants are worse off when there is an informational incumbency advantage. That is, the private incentive to acquire information relevant to the second contest is sufficiently high that contestants will increase their first period effort expenditures relative to the case of independent prizes such that they are, ex ante, worse off. The intuition behind this result is that challengers are strictly worse off than in the case of independent prizes, while the incumbent is strictly better off. Thus, contestants in the first contest stand to gain in the second contest by obtaining the prize in the first contest, and, conversely, stand to lose in the second contest by not obtaining the prize in the first contest. This added incentive is sufficient to increase aggregate effort expenditures over the two contests relative to the case of independent prizes. By way of contrast, in analogous twice-repeated all-pay and first-price auctions, expected revenue summed over both periods is unchanged between the case of an informational incumbency advantage and the case of independent values.

In the second (terminal) contest, expected effort expenditure of an individual contestant is decreasing in the number of contestants, the expected utility of a contestant is decreasing in the number of contestants, and the aggregate expected effort expenditure is increasing in the number of contestants. Interestingly, in analogous one-shot all-pay and first-price auctions, revenue and profit predictions are invariant to the number of bidders.

The second period of my model, in which the incumbent has an informational advantage, is a generalization of Wärneryd (2003), which examines a one-shot, two-player imperfectly discriminating contest where the prize is of common and uncertain

[1] Aggregate effort over the two periods strictly increases relative to the case of independent prizes if the support of distribution from which prizes are drawn includes zero.

value. My model differs in that there are $n \geq 2$ contestants, and I allow the incumbent's information to be imperfectly informative. Indeed, I assume that the value of the prize in period two is positively regression dependent on the value in period one, a weaker assumption of positive dependence than the notion of affiliated random variables used extensively throughout the auction literature.

The asymmetric information structure studied in the second contest of my model has been studied in one-shot first-price auctions by Engelbrecht-Wiggans *et al.* (1983) and Milgrom and Weber (1982). They find that this asymmetric information structure guarantees that the uninformed bidders have an expected payoff of zero. Further the informed bidder earns a positive information rent. Expected revenue is less than in a symmetric information structure due to the informed bidder's information rent. Grosskopf *et al.* (2009) considers this information structure in the context of an all-pay auction, and finds that expected revenue and the expected payoff of bidders are identical to those in a first-price auction.

This type of asymmetric information structure has also been examined in repeated games. Hörner and Jamison (2008) study an infinitely repeated first-price auction with the information structure of Engelbrecht-Wiggans *et al.* (1983). In their model, bids are observed at the end of each auction, such that uninformed bidders update their beliefs regarding the value of the good by observing the behavior of the informed bidder. Consequently, uninformed bidder are able, in finite time, to infer the informed bidder's private information.

In a paper closely related to this one, Virág (2007) examines a twice repeated first-price auction with an initial information structure as in Engelbrecht-Wiggans *et al.* (1983). There are two bidders, and one of them holds private information in the first period. Bids are not observed at the end of the period. If the uninformed bidder loses the first period auction, then asymmetric information still exists in the second period. If the uninformed bidder wins the first period auction she observes the value of the good, and information is symmetric in the second period. Virág finds that bidders bid more aggressively, as the uninformed bidder has more to gain in the first period, and the informed bidder has a higher incentive to win, in order to maintain the asymmetric information in the second period. My model differs in that contestants are symmetric (uninformed) in the first period, and I consider an imperfectly discriminating contest rather than a first-price auction. However, my results are similar to his in that, contestants expend more effort in response to the information asymmetry.

In Appendix B, I consider an incumbency advantage in which the incumbent has a strictly greater probability of obtaining the prize for any vector of effort levels. Interestingly, I find that the effect on aggregate effort expenditures over the two periods is not monotonic in the magnitude of this "status quo bias." This approach has not been considered in the literature. The closest paper is Baik and Lee (2000), which considers a contest where contestants can carry a portion of their effort in an early contest on to a final contest. They find that total effort levels increase in response to this carry-over. Their findings were generalized in Lee (2003). Schmitt *et al.* (2004) show that this kind of carry-over will not change aggregate effort in a repeated contest, although it will shift effort towards early rounds.

Also in Appendix B, I consider a model in which the incumbent enjoys a cost advantage. I find that aggregate effort expenditures increase in response to the incumbents cost advantage. In a closely related paper, Mehlum and Moene (2006) show that, if the incumbent in an infinitely repeated imperfectly discriminating contest has an inheritable cost advantage over its rival, the effort level of both contestants rises in any given period. In their model, information is complete.

The remainder of the paper is organized as follows. Section two presents the model. I first consider the case where the incumbent does not gain any useful information in $t = 1$. This is the benchmark case against which the other case is compared. The case where the incumbent has an informational advantage is then examined. Section three provides a conclusion. Alternative methods of modeling an incumbency advantage are considered in Appendix B. Namely, the case in which the incumbent has an increased probability of winning the contest for any vector of effort levels, and the case in which the incumbent has a lower marginal cost of effort than challengers.

# 2 Model

There are two periods $t = 1, 2$. In each of these periods a set of risk neutral contestants $\mathbf{N} = \{1, 2, ..., n\}$ compete for a prize with a common value. The value in period $t$ is a realization of the random variable $V_t$, where $V_1$ and $V_2$ are both distributed according to the absolutely continuous distribution function $F_V$, with support contained in $[\underline{v}, \infty)$ with $\underline{v} > 0$. The expected value of $V_t = E(V)$. This distribution function is commonly known. In period $t$ each contestant $i \in \mathbf{N}$ expends unrecoverable effort, $x_{it} \in \mathbb{R}_+$ at a cost of $C_i(x_{it}) = x_{it}$ in an effort to obtain the prize, $v_t$. These effort levels are chosen simultaneously. Contestants are not budget constrained; the strategy space of each player is $\mathbb{R}_+^2$. The vector of effort levels in period $t$ is $\mathbf{x}_t \equiv \{x_{1t}, x_{2t}, ..., x_{nt}\}$. Further, $\mathbf{x}_{-it} \equiv \mathbf{x}_t \backslash x_{it}$ and $\mathbf{N}_{-i} \equiv \mathbf{N} \backslash i$.

The function $p_{it} : \mathbb{R}_+^n \rightarrow [0,1]$ maps $\mathbf{x}_t$ into the probability that contestant $i$ will receive the good in period $t$. This function, which is typically called the contest success function in the contest literature, is given by

$$p_{it}(x_{it}, \mathbf{x}_{-it}) = \begin{cases} \frac{x_{it}}{x_{it} + \sum\limits_{j \in \mathbf{N}_{-i}} x_{jt}} & \text{if} \quad \max \mathbf{x}_t \neq 0 \\ b_i & \text{if} \quad \max \mathbf{x}_t = 0, \end{cases}$$

where $b_i \in [0,1]$ for any $i$ and $\sum_{i \in \mathbf{N}} b_i \leq 1$. Note that $b_i$ is the probability that player $i$ receives $v_t$ when none of the contestants expend positive effort in $t$. Different applications suggest different assumptions regarding $\mathbf{b} \equiv \{b_1, b_2, ..., b_n\}$. Two common assumptions are $b_i = \frac{1}{n}, \forall i \in \mathbf{N}$ or that $b_i = 0, \forall i \in \mathbf{N}$. The choice of $\mathbf{b}$ does not affect the following results. This contest success function is a special case of the class axiomized in Skaperdas (1996) and defines what is sometimes called a lottery contest because the probability of a contestant obtaining the prize is her proportion of total effort, as in a lottery.

Contestants in period $t$ do not observe the value of $v_t$ before choosing $x_{it}$. At the conclusion of period $t$, one of the contestants receives the prize, and privately observes $v_t$. As such, before contestants choose their effort expenditures in $t = 2$ the contestant who received the good in $t = 1$ (the incumbent) holds private information, while the remaining contestants (the challengers) hold only public information. The incumbent is denoted as contestant $I$. The set of contestants who did not obtain the prize in $t = 1$, the challengers, is $\mathbf{C} \equiv \mathbf{N}/I$. $\mathbf{C}_{-j} \equiv \mathbf{N}_{-j} \cap \mathbf{C}$ is the set of challengers that does not include contestant $j$ and $\mathbf{x}_C \equiv \{x_{j2} : j \in \mathbf{C}\}$ is the vector of effort levels chosen by the challengers.

## 2.1 Intertemporal Independence of Values (IIV)

Consider the case in which $v_1$ and $v_2$ are independent draws from $F_V$. In this case $E(V_2 \mid v_1) = E(V)$; the incumbent's private observation of $v_1$ does not provide information of strategic importance in $t = 2$. Thus, this game is a twice repeated contest in which the outcome in $t = 1$ does not affect the symmetry of contestants in $t = 2$. This case provides a benchmark against which incumbency advantages can be compared.

The analysis of the incumbents problem is identical to that of a challenger. The analysis begins in $t = 2$, where contestant $i$'s expected utility is

$$U_{i2}^{IIV} \equiv \int_{\underline{v}}^{\infty} p_{i2}(x_{i2}, \mathbf{x}_{-i2}) v_2 dF_V(v_2) - x_{i2}.$$

This objective function is strictly concave in $x_{i2}$ given $\mathbf{x}_{-it}$, so the first order condition defines a best response. This first order condition is

$$\frac{E(V) \sum_{j \in \mathbf{N}_{-i}} x_{j2}}{\left( x_{i2} + \sum_{j \in \mathbf{N}_{-i}} x_{j2} \right)^2} - 1 = 0.$$

Note that there is no best response to $\sum_{j \in \mathbf{N}_{-i}} x_{j2} = 0$; for any $x_{i2} > 0$ contestant $i$ obtains the prize with certainty, but has an incentive to reduce $x_{i2}$ to a smaller positive number. As such, the best response function of contestant $i$ is well defined on the interval $(0, \infty)$, and is given by

$$x_{i2}(\mathbf{x}_{-i2}) = \begin{cases} \sqrt{\sum_{j \in \mathbf{N}_{-i}} x_{j2} E(V)} - \sum_{j \in \mathbf{N}_{-i}} x_{j2} & \text{if } \sum_{j \in \mathbf{N}_{-i}} x_{j2} \in (0, E(V)] \\ 0 & \text{if } \sum_{j \in \mathbf{N}_{-i}} x_{j2} \in (E(V), \infty). \end{cases}$$

The well-known, unique[2] equilibrium is symmetric, and $\forall i \in \mathbf{N}$ expends

$$x_{i2}^{IIV} \equiv \frac{E(V)(n-1)}{n^2}.$$

Note that $x_{i2}^{IIV}$ is decreasing in $n$, and limits to zero. Denoting equilibrium aggregate effort expenditures in period $t$ of the $IIV$ case as $R_t^{IIV}$, $R_2^{IIV} = \sum_{i \in \mathbf{N}} x_{i2}^{IIV} =$

[2] See, for example, Yamazaki (2008).

$\frac{E(V)(n-1)}{n}$ which is strictly less than $E(V)$ and increasing in $n$. Note that $\lim_{n\to\infty} R_2^{IIV} = E(V)$. The aggregate effort expenditures in imperfectly discriminating contests is often referred to as rent dissipation, a reference to rent seeking applications in which effort expenditures are a social bad.

The equilibrium expected utility of contestant $i$ in $t = 2$ is

$$E\left(U_{i2}^{IIV}\right) = \int_{\underline{v}}^{\infty} \frac{x_{i2}^{IIV}}{\sum_{j\in\mathbf{N}} x_{j2}^{IIV}} v_2 dF_V(v_2) - x_{i2}^{IIV} = \frac{E(V)}{n^2}.$$

Note that $E\left(U_{i2}^{IIV}\right)$ is decreasing in $n$ and that $\lim_{n\to\infty} E\left(U_{i2}^{IIV}\right) = 0$. Contestants have positive expected utility, despite not holding any private information. This is attributable to the functional form of the contest success function, in which the highest effort level does not win with certainty, which induces contestants to expend less effort than $E(V)$. Since this equilibrium is symmetric, each of the contestants has an equal chance of obtaining the prize.

In $t = 1$ contestant $i$'s expected utility is

$$U_{i1}^{IIV} \equiv \int_{\underline{v}}^{\infty} p_{i1}(x_{i1}, \mathbf{x}_{-i1}) v_1 dF_V(v_1) - x_{i1} + E\left(U_{i2}^{IIV}\right).$$

Since $E\left(U_{i2}^{IIV}\right)$ does not depend on $\mathbf{x}_t$ or $v_1$, strategic considerations in $t = 1$ are identical to those in $t = 2$, and the equilibrium effort of contestant, $x_{i1}^{IIV}$ is identical to that found in $t = 2$. That is, $x_{i1}^{IIV} = x_{i2}^{IIV}$, which also implies that $R_1^{IIV} = R_2^{IIV}$ and that $E\left(U_{i1}^{IIV}\right) = E\left(U_{i2}^{IIV}\right)$. Further, each of the contestants has an equal chance of obtaining the prize.

The sum of equilibrium effort expenditures across $t = 1, 2$ , is

$$R^{IIV} \equiv \sum_{t=1}^{2} R^{IIV} = \frac{2E(V)(n-1)}{n}. \tag{1}$$

Note that $R^{IIV}$ is increasing in $n$. Further $\lim_{n\to\infty} E\left(U_{i2}^{IIV}\right) = 2E(V)$.

Notice that if contestants are know the value of the prize in either or both contests prior to choosing their effort expenditures, the ex ante results are unchanged. In particular, if all contestants are informed of $v_t$, it is easy to show that the equilibrium effort level, $x_{it}^{INF}$, is

$$x_{it}^{INF} = \frac{v_t\left(n-1\right)}{n^2}.$$

But note that $E\left(x_{it}^{INF}\right) = x_{it}^{IIV}$. Thus, ex ante, the equilibrium predictions of the IIV case are identical to the case in which contestants are symmetrically informed.

## 2.2 Intertemporal Dependence of Values (IDV)

Consider the case in which $V_2$ is positively regression dependent on $V_1$. Positive regression dependence dictates that $P\left(V_2 \leq v_2 \mid V_1 = v_1\right)$ be non-increasing in $v_1$ for all $v_2$.[3] Intuitively, positive regression dependence implies as $v_1$ increases, the probability that $V_2$ will be large increases. Positive regression dependence is a strictly weaker concept of positive dependence than affiliated random variables, which is used extensively in auction theory; affiliation implies positive regression dependence.[4] Thus, the following results are also implied by affiliation between $V_1$ and $V_2$. Recall that the marginal distributions of $V_1$ and $V_2$ are identical, and equal to $F_V$. $V_1$ and $V_2$ are jointly distributed with the joint density function $f\left(v_1, v_2\right)$. The absolutely continuous joint distribution function of these random variables is $F\left(v_1, v_2\right)$. The distribution function of $V_2$, conditional on $V_1$, is $F\left(v_2 \mid v_1\right)$. Since $V_2$ is positively regression dependent on $V_1$, $F\left(v_2 \mid v_1\right)$ is non-increasing in $v_1$ for any $v_2$. To ensure that $E\left(V_2 \mid v_1\right)$ is strictly increasing in $v_1$, I assume that for $v_1' > v_1$, $F\left(v_2 \mid v_1'\right) < F\left(v_2 \mid v_1\right)$ for at least one $v_2 \in [\underline{v}, \infty)$.

In $t = 2$ the incumbent has observed $v_1$, which provides information regarding $v_2$ in the form of $E\left(V_2 \mid v_1\right)$. This introduces asymmetric information into the contest in

[3]See Lehmann (1966).

[4]For proof of this implication, see Yanagimoto (1972). This is also shown in de Castro (2007).

$t = 2$; the incumbent holds private information which allows her to form an updated expectation regarding $v_2$, while the challengers hold only public information. The information structure of the subgame in $t = 2$ is studied in Wärneryd (2003), with $n = 2$ and a perfectly informed contestant. What follows generalizes those results since the informed contestant (the incumbent) need not be perfectly informed of $v_2$ and there are $n \geq 2$ contestants.

As above, the incumbent is denoted as contestant $I$. The set of contestants who did not win the prize in $t = 1$, the challengers, is $\mathbf{C} \equiv \mathbf{N}/I$. $\mathbf{C}_{-j} \equiv \mathbf{N}_{-j} \cap \mathbf{C}$ is the set of challengers that does not include contestant $j$ and $\mathbf{x}_C \equiv \{x_{j2} : j \in \mathbf{C}\}$ is the vector of effort levels chosen by the challengers. The incumbent's expected utility now depends on the privately observed $v_1$, and is given by

$$U_{I2}^{IDV}(v_1) \equiv \int_{\underline{v}}^{\infty} p_{I2}(x_{I2}(v_1), \mathbf{x}_C)\, v_2 dF(v_2 \mid v_1) - x_{I2}(v_1).$$

This expected utility is strictly concave in $x_{I2}(v_1)$, given $\mathbf{x}_C$ such that the first order condition is sufficient to establish a maximum. The partial derivative with respect to $x_{I2}(v_1)$ is

$$\frac{\sum\limits_{j \in \mathbf{C}} x_{j2}}{\left(x_{I2}(v_1) + \sum\limits_{j \in \mathbf{C}} x_{j2}\right)^2} E(V_2 \mid v_1) - 1.$$

Any $x_{I2}(v_1) \geqslant 0$ renders this expression negative if $\sum_{j \in \mathbf{C}} x_{j2} > E(V_2 \mid v_1)$. Thus, if the summed effort of the challengers is greater than the incumbent believes the prize is worth, the incumbent's best response is to expend no effort.. If $\sum_{j \in \mathbf{C}} x_{j2} \leq E(V_2 \mid v_1)$ then there exists a $x_{I2}(v_1) \geqslant 0$ for which the partial derivative is equal to zero. Since $E(V_2 \mid v_1)$ is strictly monotonically increasing in $v_1$, $\sum_{j \in \mathbf{C}} x_{j2} \leq E(V_2 \mid v_1)$ will hold with equality for exactly one $v_1$ if $\sum_{j \in \mathbf{C}} x_{j2} \geq E(V_2 \mid \underline{v})$. Thus, if $\sum_{j \in \mathbf{C}} x_{j2} \geq E(V_2 \mid \underline{v})$, then the expression $\sum_{j \in \mathbf{C}} x_{j2} = E(V_2 \mid v_1)$ defines a threshold value of $v_1$ above which the incumbent will expend positive effort. Since of $E(V_2 \mid v_1)$ is monotonic in $v_1$, its inverse, $s(\cdot)$, is well defined on $[E(V_2 \mid \underline{v}), \infty)$, and the threshold value of $v_1$ that the challenger must observe in order for $x_{I2}(v_1) \geq 0$

to be a best response to $\sum_{j\in\mathbf{C}} x_{j2}$ is

$$q\left(\sum_{j\in\mathbf{C}} x_{j2}\right) \equiv \begin{cases} s\left(\sum\limits_{j\in\mathbf{C}} x_{j2}\right) & \text{if} \quad \sum\limits_{j\in\mathbf{C}} x_{j2} \geq E\left(V_2 \mid \underline{v}\right) \\ \underline{v} & \text{if} \quad \sum\limits_{j\in\mathbf{C}} x_{j2} < E\left(V_2 \mid \underline{v}\right). \end{cases}$$

The best response function of the incumbent, which is defined on the domain $(0, \infty)$, can then be expressed as

$$x_{I2}\left(v_1\right) = \begin{cases} \sqrt{\sum\limits_{j\in\mathbf{C}} x_{j2} E\left(V_2 \mid v_1\right)} - \sum\limits_{j\in\mathbf{C}} x_{j2} & \text{if} \quad q\left(\sum\limits_{j\in\mathbf{C}} x_{j2}\right) \leq v_1 \\ 0 & \text{if} \quad q\left(\sum\limits_{j\in\mathbf{C}} x_{j2}\right) > v_1. \end{cases}$$

In equilibrium, the ex ante expected effort expenditure of the IDV incumbent is denoted as $E\left(x_{I2}^{IDV}\left(V_1\right)\right)$.

The expected utility of contestant $j \in \mathbf{C}$ is

$$U_{j2}^{IDV} \equiv E\left(\int_{\underline{v}}^{\infty} \frac{x_{j2} V_2}{x_{I2}\left(V_1\right) + x_{j2} + \sum\limits_{k\in\mathbf{C}_{-j}} x_{k2}}\right) - x_{j2}.$$

As before, the strict concavity of this objective function in $x_{j2}$ given $\mathbf{x}_{-it}$ implies that the first order condition yields a maximum. This first order condition is

$$E\left(\frac{\left(x_{I2}\left(V_1\right) + \sum\limits_{k\in\mathbf{C}_{-j}} x_{k2}\right) v_2}{\left(x_{I2}\left(V_1\right) + x_{j2} + \sum\limits_{k\in\mathbf{C}_{-j}} x_{k2}\right)^2}\right) - 1 = 0.$$

The $(n-1)$ challengers each expend the same quantity of effort in equilibrium. To see this, consider the case in which contestant $m \in \mathbf{C}$ optimally expends $x_{m2} > 0$

while contestant $l \in \mathbf{C}$ optimally expends $x_{l2} > x_{m2}$ Since $x_{l2} > x_{m2} > 0$ the first order conditions for contestants $l$ and $m$ hold with equality such that

$$E\left(\frac{\left(x_{I2}\left(V_1\right)+\sum_{k\in\mathbf{C}/\{l,m\}}x_{k2}+x_{l2}\right)v_2}{\left(x_{I2}\left(V_1\right)+\sum_{k\in\mathbf{C}}x_{k2}\right)^2}\right)=E\left(\frac{\left(x_{I2}\left(V_1\right)+\sum_{k\in\mathbf{C}/\{l,m\}}x_{k2}+x_{m2}\right)v_2}{\left(x_{I2}\left(V_1\right)+\sum_{k\in\mathbf{C}}x_{k2}\right)^2}\right).$$

But this is a contradiction since $x_{l2} > x_{m2}$. Thus, if challengers are optimally expending a positive amount of effort, they each expend the same amount of effort. Likewise, the case in which one of the challengers is optimally expending zero effort implies that this is the optimal choice for the remaining challengers as well. The $(n-1)$ challengers can not expend zero effort in an equilibrium, since the best response of the incumbent does not exist when $\sum_{j\in\mathbf{C}} x_{j2}$.

The equilibrium effort of a challenger in the IDV case is denoted by $x_{C2}^{IDV}$, and the sum of the challengers' effort expenditures is equal to $x_{C2}^{IDV}(n-1)$. Utilizing the incumbent's best response function simplifies the first order condition of a challenger. The resulting equation relates the equilibrium effort level of a challenger to the expected equilibrium effort level of the incumbent, where $\mathbf{1}_B$ is the indicator function that is equal to one if $B$ is true, and zero otherwise,

$$\begin{aligned} x_{C2}^{IDV} &= \left(\frac{1}{\left(1+F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\left(n-2\right)\right)}\right)E\left(x_{I2}^{IDV}\left(V_1\right)\right) \\ &\quad+\left(\frac{n-2}{\left(n-1\right)\left(1+F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\left(n-2\right)\right)}\right)E\left(V_2\mathbf{1}_{V_1\le q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned} \tag{2}$$

Note that this is not a closed form solution for $x_{C2}^{IDV}$, as it appears on both sides of the equation. Plugging in the best response function of the incumbent and simplifying (2) further yields the following equation, which characterizes equilibrium in $t = 2$

$$\begin{aligned} & n-F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \\ =\; & \left(\frac{(n-2)}{x_{C2}^{IDV}\left(n-1\right)}\right)E\left(V_2\mathbf{1}_{V_1\le q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ & +\sqrt{\frac{(n-1)}{x_{C2}^{IDV}}}E\left(\sqrt{E\left(V_2\mid V_1\right)}\mathbf{1}_{V_1\ge q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned} \tag{3}$$

Consider the special case where $x_{C2}^{IDV}(n-1) < E(V_2 \mid \underline{v})$. In this case there is no $v_1$ for which the incumbent believes the challengers are expending more effort than the prize is worth and $x_{I2}(v_1) > 0$ for any $v_1$. Following Wärneryd (2003), I call this an interior equilibrium. In such a situation (2) and (3) become

$$E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV},$$

$$x_{C2}^{IDV} = \frac{(n-1)\left(E\left(\sqrt{E(V_2 \mid V_1)}\right)\right)^2}{n^2}.$$

Thus, if $x_{C2}^{IDV}(n-1) \leq E(V_2 \mid \underline{v})$, then there is an explicit solution for the equilibrium of this subgame. A sufficient condition for the existence of such an interior equilibrium is

$$\left(\frac{(n-1)\,E\left(\sqrt{E(V_2 \mid V_1)}\right)}{n}\right)^2 \leq E(V_2 \mid \underline{v}).$$

This sufficient condition restricts attention to a narrow set of distribution functions, and a more general result is desirable.

If $x_{C2}^{IDV}(n-1) > E(V_2 \mid \underline{v})$ the incumbent does not expend positive effort for some realizations of $v_1$. Consequently, there is no closed form solution for equilibrium. Furthermore, since the best response function of the incumbent is not defined at $\sum_{j \in \mathbf{C}} x_{j2} = 0$, the Banach fixed point theorem cannot be utilized to guarantee the existence or uniqueness of equilibrium in this subgame. However, the following result holds.

**Proposition 1** *There is a unique Nash equilibrium in $t = 2$ of the IDV case.*

**Proof.** See Appendix A. ■

If the unique equilibrium is interior, then $E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV}$, and $x_{I2}^{IDV}(V_1) > 0$, for all $v_1$. When the equilibrium is not interior, there are values of $v_1$ for which the incumbent will not expend any effort, which might suggest that a lack of an interior equilibrium would depress $E\left(x_{I2}^{IDV}(V_1)\right)$ relative to $x_{C2}^{IDV}$. Accordingly, the expected effort expenditure of the incumbent relative to a challenger is of interest. The following result refutes the line of thinking outlined above.

**Proposition 2** *In the IDV case, the ex ante expected effort expenditure of the incumbent is weakly greater than that of a challenger. If $n = 2$, or there is an interior equilibrium, the incumbent's ex ante expected effort level is equal to that of a challenger, otherwise the inequality is strict.*

**Proof.** See Appendix A. ■

The intuition behind this result relies on the fact that the incumbent's best response function is increasing in $v_1$; she expends less effort than a challenger when $v_1$ is low, and more when $v_1$ is high. Consequently, a challenger is more likely to obtain the prize when $v_1$ is low, so that the expected value of the prize conditional on having been obtained by a challenger is lower than $E(V)$. Challengers reduce their effort expenditures relative to the incumbent to account for this. When the equilibrium is not interior incumbents do not expend any effort for low values of $v_1$ so that a challenger obtains the prize with certainty, providing challengers a stronger incentive to reduce their effort expenditures than in an interior equilibrium. That is, the presence of asymmetric information introduces a winner's curse for challengers, in which obtaining the prize depresses a challengers beliefs regarding its worth. A similar winner's curse arises in a first-price, sealed-bid auction with the $t = 2$ IDV information structure.[5]

The lottery contest success function utilized in this model awards the prize to a contestant with probability equal to her proportion of aggregate effort expenditures in the contest. Since $E\left(x_{I2}^{IDV}(V_1)\right) \geq x_{C2}^{IDV}$, the incumbent has, ex ante, the (weakly) highest proportion of aggregate effort. Recall that $E\left(x_{I2}^{IDV}(V_1)\right) > x_{C2}^{IDV}$ when the equilibrium in not interior and $n = 2$. As such, the following result is somewhat counterintuitive.

In equilibrium the incumbent will expend more effort than a challenger when $v_1$ is high, such that, ex ante, she is expected to expend more than a challenger, despite

[5] See Engelbrecht-Wiggans et al. (1983) and Milgrom and Weber (1982).

choosing $x_{I2}(v_1) = 0$ if $v_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)$. Further, in equilibrium the incumbent obtains the prize with positive probability only when $v_1 > q\left(x_{C2}^{IDV}(n-1)\right)$. Thus, there are two effects influencing the ex ante probability of the incumbent obtaining the good in $t = 2$. I find the following, which holds in an interior equilibrium as well.

**Proposition 3** *In the IDV case the incumbents ex ante expected probability of obtaining the prize is strictly less than that of a challenger.*

**Proof.** See Appendix A. ■

This result yields an interesting insight into the effect of an informed incumbent. In particular, incumbents are less entrenched under this informational asymmetry than in the IIV case; the incumbent is ex ante less likely to obtain the prize in $t = 2$. The intuition is that, in equilibrium the incumbent obtains the prize with positive probability only when $v_1 > q\left(x_{C2}^{IDV}(n-1)\right)$. Further, since the incumbent only expends more effort than a challenger when $x_{C2}^{IDV} \leq (n-1)E(V_2 \mid v_1)/n^2$, a challenger will obtain the prize with high probability when $v_1$ is low.

Contrasting this result with the analogous findings in standard auction formats is worthwhile. As mentioned above, the information structure in $t = 2$ of the IDV case has been studied in the context of first-price sealed-bid auctions in Engelbrecht-Wiggans *et al.* (1982) and in the context of all-pay auctions in Grosskopf *et al.* (2009). In both of these auction formats, the ex ante probability that the informed bidder wins the auction is 50%, regardless of the number of bidders.

To ascertain the effect of the assumption that $V_2$ is positive regression dependant on $V_1$, consider the equilibrium effort expenditure of challengers in the IDV case to that of contestants in $t = 2$ of the IIV case. If the equilibrium is interior, notice that Jensen's Inequality yields

$$\begin{aligned}
E\left(x_{I2}^{IDV}\left(V_1\right)\right) &= x_{C2}^{IDV} \\
&= \frac{(n-1)\left(E\left(\sqrt{E\left(V_2 \mid V_1\right)}\right)\right)^2}{n^2} \\
&< \frac{(n-1)\,E\left(V\right)}{n^2} \\
&= x_{i2}^{IIV}.
\end{aligned}$$

Since $E\left(x_{I2}^{IDV}\left(V_1\right)\right) = x_{C2}^{IDV} < x_{i2}^{IIV}$, the expected revenue of such an interior equilibrium, $R_2^{IDV}$, is strictly less than $R_2^{IIV}$. A more general result follows.

**Proposition 4** *The equilibrium effort expenditure of a contestant in $t = 2$ of the IIV case is strictly greater than the equilibrium effort expenditure of a challenger in $t = 2$ of the IDV case if and only if*

$$\begin{aligned}
&\frac{(n-2)}{(n-1)\left(n - F_V\left(q\left(B\right)\right)\right)} E\left(V_2 1_{V_1 \le q(B)}\right) \\
&+ \frac{(n-1)\sqrt{E\left(V\right)}}{n\left(n - F_V\left(q\left(B\right)\right)\right)} E\left(\sqrt{V_2} 1_{V_1 \ge q(B)}\right) \\
< &\ \frac{E\left(V\right)(n-1)}{n^2},
\end{aligned} \tag{4}$$

*where $B \equiv \frac{E(V)(n-1)^2}{n^2} = x_{i2}^{IIV}\left(n-1\right)$.*

**Proof.** See Appendix A. ■

Equation (4), which holds trivially when $n = 2$, states that if the IDV incumbent were to best respond to the equilibrium strategy of the challengers in the IIV case ($\sum_{j \in \mathbf{C}} x_{j2} = x_{i2}^{IIV}\left(n-1\right)$), then the best response of the IDV challengers is to reduce their effort expenditures relative to the IIV case. Suppose the IDV challengers expend $\sum_{j \in \mathbf{C}} x_{j2} = x_{i2}^{IIV}\left(n-1\right)$. Since, in this scenario, the incumbent's equilibrium effort expenditure is monotonically increasing in $v_1$ when $v_1 \ge q\left(x_{i2}^{IIV}\left(n-1\right)\right)$, and she expends more effort than $x_{i2}^{IIV}$ only when $x_{i2}^{IIV} \le \left(n-1\right)E\left(V_2 \mid v_1\right)/n^2$, a challenger who expends $x_{i2}^{IIV}$ is more likely to obtain the prize when it has a low value. As discussed above, the expected value of the prize, conditional on a challenger having obtained it, is then less than $E(V)$. As such, it is reasonable to assume that risk-neutral challengers shade their effort levels below $x_{i2}^{IIV}$, as required by (4). It is important to note that (4) is not a restrictive assumption; for example the Pareto, Gamma, Uniform and Triangular distributions all satisfy if for a broad range of parameterizations. In what follows, I assume that (4) is satisfied.

Interestingly, the comparison between $E\left(x_{I2}^{IDV}(V_1)\right)$ and $x_{i2}^{IIV}$ depends on $n$ and the distribution function $F_V$. If there is an interior equilibrium or if $n = 2$, then $E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV} < x_{i2}^{IIV}$. When the equilibrium in not interior and $n > 2$, the incumbent expends $E\left(x_{I2}^{IDV}(V_1)\right) > x_{i2}^{IIV}$ when

$$\begin{aligned} x_{i2}^{IIV} - x_{C2}^{IDV} &< (n-2)\, x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) \\ &\quad - \frac{(n-2)}{(n-1)} E\left(V_2 1_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

Since the equilibrium is not interior if $E\left(x_{I2}^{IDV}(V_1)\right) > x_{i2}^{IIV}$, and there is no closed form solution for such an equilibrium, I am unable to give further conditions. However, examples demonstrate that $E\left(x_{I2}^{IDV}(V_1)\right) > x_{i2}^{IIV}$ in many cases. For example, if $V_1 = V_2 \sim U(1, 11)$, and $n = 200$, then $E\left(x_{I2}^{IDV}(V_1)\right) = 0.79$, while $x_{i2}^{IIV} = 0.03$.

As mentioned above, the information structure in $t = 2$ of the IDV case has been studied in a variety of auction formats. Engelbrecht-Wiggans *et al.* (1983) finds that this asymmetric information structure guarantees that the uninformed bidders have expected payoff of zero in any equilibrium of any standard auction format. Further, in all-pay and first-price auctions, the informed bidder earns a positive information rent. Since the expected payoff off bidders in the symmetric information structure in which no bidders hold private information is zero (as in the IIV case), this information rent is extracted from the seller.[6]

Comparing the ex ante expected utility of contestants in $t = 2$ of the IIV and IDV cases is of interest as it reveals the effect of information asymmetry. Additionally, comparing these results to those found in all-pay and first-price auctions yields insight into the effect of utilizing an imperfectly discriminating contest success function. Note that the expected utility of a contestant in $t = 2$ of the IIV case is $E\left(U_{i2}^{IIV}\right) =$

[6]See Grosskopf et al. (2009) and Milgrom and Weber (1982).

$E(V)/n^2 > 0$, whereas in the analogous first-price or all-pay auction her expected utility would be zero.[7] This is attributable to the imperfectly discriminating nature of the lottery contest considered.

Notice that, in equilibrium, the expected utility of a challenger in the IDV case can be written as

$$\begin{aligned} E\left(U_{C2}^{IDV}\right) &= \frac{1}{(n-1)^2} E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ &+ \frac{x_{C2}^{IDV}\left(1 - F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)\right)}{(n-1)}. \end{aligned}$$

Since $x_{C2}^{IDV} > 0$ in equilibrium, $E\left(U_{C2}^{IDV}\right) > 0$. Since the expected utility of uninformed bidders in all-pay auctions is zero, the imperfectly discriminating contest success function allows IDV challengers to earn a positive expected utility, despite the information asymmetry. While the presence of asymmetric information does not reduce $E\left(U_{C2}^{IDV}\right)$ to zero, I have the following result.

**Proposition 5** *If (4) is satisfied, then the ex ante expected utility of a challenger is strictly less in the IDV case than the IIV case.*

**Proof.** See Appendix A. ■

In contrast to the aforementioned results in all-pay and first-price auctions, an information asymmetry makes the challengers worse off. Notice that while bidders who do not observe a signal regarding the value of the good in an all-pay or first price auction are indifferent between the information structures in $t = 2$ of the IDV and IIV case, the same is not true in the lottery contest.

Next, I look at the expected utility of the incumbent. Utilizing (3) and the best response function of the incumbent, the ex ante equilibrium expected utility of the incumbent can be written as

$$\begin{aligned} E\left(U_{I2}^{IDV}\right) &= E(V) + \frac{(n-3)}{(n-1)} E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ &- x_{C2}^{IDV}\left((n+1) + F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-3)\right). \end{aligned}$$

[7] See Baye (1996) for an analysis of all-pay auctions under complete information.

I can now say the following.

**Proposition 6** *If (4) is satisfied the ex ante expected utility of the incumbent is strictly greater in the IPV case than in the IIV case.*

**Proof.** See Appendix A. ■

The IDV incumbent earns a positive information rent. Since the challengers are ex ante worse off in the IDV case, at least some of this information rent is extracted from them. The effect of the information asymmetry on aggregate effort expenditures in $t = 2$ of the IDV case is closely related since for any $v_1$, it must be the case that the sum of effort expenditures and realized payoffs of the contestants equal $E(V_2 \mid v_1)$. In expectation, $E\left(U_{I2}^{IDV}\right) + E\left(U_{C2}^{IDV}\right)(n-1) + R_2^{IDV} = E(V)$. As such, $E\left(U_{I2}^{IDV}\right) + E\left(U_{C2}^{IDV}\right) > 2E\left(U_{i2}^{IIV}\right)$, would indicate that $R_2^{IDV} < R_2^{IIV}$. The following result establishes this.

**Proposition 7** *If (4) is satisfied, ex ante expected effort expenditures are strictly lower in the IDV case than in the IIV case.*

**Proof.** See Appendix A. ■

This result shows that the information rent earned by the IDV incumbent is extracted from the challengers, and by reducing aggregate effort expenditures in $t = 2$. The ex ante expected value of obtaining the prize in $t = 1$ is then $E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right) > E(V)$. Thus, contestants in $t = 1$ of the IDV case have an increased incentive to obtain the prize.

In $t = 1$ the $n$ contestants are symmetric. None of them hold private information, although they are aware that privately observing $v_1$ will, in expectation earn them an information rent. The expected utility of contestant $i$ in $t = 1$ is:

$$\begin{aligned} U_{i1}^{IDV} &\equiv p_{i1}\left(x_{I2}(v_1), \mathbf{x}_C\right)\left(E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right) \\ &\quad -x_{i1} + E\left(U_{C2}^{IDV}\right) \end{aligned}$$

This problem is strategically equivalent to a complete information contest with a prize of $E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)$. As in the IIV case, there is a unique equilibrium which is symmetric. The equilibrium effort expenditure of contestant $i$ in $t = 1$ is

$$x_{i1}^{IDV} \equiv \frac{\left(E\left(V\right) + \left(E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right)\right)\left(n-1\right)}{n^2}$$

$\forall i \in \mathbf{N}$.

Since $E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right) > 0$, $x_{i1}^{IDV} > x_{i1}^{IIV}$, which implies that $R_1^{IDV} > R_1^{IIV}$. The sum of ex ante expected effort expenditures across both periods is $R^{IDV} \equiv \sum_{t=1}^{2} R_t^{IDV}$. Since, $R_2^{IDV} < R_2^{IIV}$, the effect of the information asymmetry on total effort expenditures across the two periods is of interest.

**Proposition 8** *When the equilibrium in $t = 2$ of the IDV case is not interior , total effort expenditures in the IPV case, $R^{IDV}$, strictly exceed those of the IIV case. If the equilibrium in $t = 2$ of the IDV case is interior then $R^{IDV} = R^{IIV}$.*

**Proof.** See Appendix A. ■

It is worth noting that if the game were modified such that in $t = 1$, contestants were to compete for the chance to privately observe $v_1$ without obtaining it, that this result holds. That is, if the contest in $t = 1$ is over the acquisition of information, the result is the same. Notice that if $\underline{v} = 0$, then there can not be an interior equilibrium, and $R^{IDV} > R^{IIV}$. Since $R^{IDV} \geq R^{IIV}$, the reduction of effort expenditures in $t = 2$ of the IDV case, are at least offset by the increase in effort expenditures in $t = 1$. Interestingly, in a twice repeated first-price or all-pay auction, analogous to the IIV and IDV cases studied here, revenue summed across the two periods is, ex ante, unchanged between the two information structures. The intuition is that in $t = 2$ of an IDV information structure the uninformed bidders earn an expected payoff of zero, while the informed bidder earns a positive information rent. In $t = 1$ the value of winning the auction is this information rent plus $E\left(V\right)$. The revenue $t = 1$ is equal to this value, because the game in $t = 1$ is a complete information auction in which the equilibrium expected utility is equal to zero.

Further, $R^{IDV} \geq R^{IIV}$ implies that the ex ante expected utility of a contestant in $t = 1$ of the IDV case is (weakly) less than in the IIV case. As such, if a contestant were offered the choice between the information structures in the IDV and IIV case, she would weakly prefer the IIV case.

Recall that as $n$ increases in the IIV case, aggregate effort expenditures increase in both periods, and so, overall. Likewise, the equilibrium expected utility of contestants is decreasing in $n$ in both periods and overall. Also, the equilibrium probability of obtaining the prize in each period, $1/n$, is decreasing in $n$ as well. Consider the effect of an increase in $n$ on behavior in the IDV case. If the equilibrium in $t = 2$ of the IDV case is not interior, then the equilibrium is characterized by the implicit function (3). Totally differentiating (3) yields the following result.

**Proposition 9** *The equilibrium effort expenditure of a challenger and of the incumbent in the IDV case is decreasing in $n$. The ex ante expected aggregate effort expenditures in $t = 2$ is increasing in $n$.*

**Proof.** See Appendix A. ■

In $t = 1$ of the IDV case the equilibrium is analogous to that of the IIV case, except with an expected value of obtaining the prize equal to $E(V) + \left(E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right)$. It is therefore straightforward to show that the comparative statics in $t = 1$ of the IDV case are consistent with those of the IIV case.

Contrasting this result with all-pay, first-price and second-price auctions reveals significant differences. In an asymmetric information structure as in IDV case, equilibrium bidding strategies and revenue predictions are invariant to the number of bidders in first-price, all-pay and second-price auctions. In an imperfectly discriminating contest, this is not the case.

Another interesting exercise is to vary the level of positive dependence between $V_1$ and $V_2$. The value of information has garnered considerable attention in the literature, mostly in the context of decision problems.[8] These results do not generalize to games, although the value of information in zero sum games has been, dealing

[8] See Blackwell (1953).

with a finite partition of the state space has been studied. Unfortunately, this setup does not directly apply to this model.

However, the fact that there is a unique Nash equilibrium for the contest with asymmetric information suggests that comparing equilibrium payoffs under different information structures may yield results. Consider two information structures, defined by their joint distribution functions: $F(v_1, v_2)$ and $G(v_1, v_2)$ where these two distribution functions have identical marginals, namely $F_V$. Kimeldorf and Sampson (1987) say that $G(v_1, v_2)$ is more positively quadrant dependent than $F(v_1, v_2)$ if $G(v_1, v_2) \geq F(v_1, v_2)$ for all $(v_1, v_2) \in \mathbb{R}^2$. In this positive dependence ordering, $V_1$, $V_2$ are more positively dependent under $G(v_1, v_2)$ than $F(v_1, v_2)$. Since the equilibrium need not be interior, comparing the equilibria under $G(v_1, v_2)$ than $F(v_1, v_2)$ yields ambiguous results. As such I am unable to give a general result regarding the affect of changes in the quality of signal.

I next introduce an example in which there is a particularly tractable way to vary the informativeness of $v_1$. In this example, $n = 2$, and the value of the prize in period two is uniformly distributed on $[\underline{v}, \overline{v}]$. It is also assumed that $\overline{v} < 7\underline{v}$. Let a second random variable, $E$, be uniformly distributed on $[-\delta, \delta]$, with $\delta > 0$. To ensure that $\delta$ is not so high as to render the signal devoid of information, it is also assumed that $\delta < \overline{v} - \underline{v}$. The signal that the incumbent receives is then $V_1 = V_2 + E$. Thus, the signal received by the incumbent must be within $\delta$ of the actual value of the prize. Examining how equilibrium effort changes in response to changes in $\delta$ is equivalent to observing the effect of changes in signal quality on equilibrium effort. Note that this example is not consistent with the model outlined above in that the distribution of $V_1$ is not the same as the distribution of $V_2$. However, it does yield some insight into how the quality of information affects equilibrium effort levels. Since $\overline{v} < 7\underline{v}$, the equilibrium is interior equilibrium. The closed form of this equilibrium is

$$\begin{aligned} x_{C2}^{IDV} &= \frac{\left(E\left(\sqrt{E\left(V_2 \mid v_1\right)}\right)\right)^2}{4} \\ &= \frac{2\left(4\underline{v}^{\frac{5}{2}} + 4\overline{v}^{\frac{5}{2}} - 4\overline{v}^2\sqrt{\overline{v}-\delta} + 3\overline{v}\sqrt{\overline{v}-\delta}\delta\right)}{15\left(\overline{v}-\underline{v}\right)\delta} \\ &\quad + \frac{2\left(4\underline{v}^2\sqrt{\underline{v}+\delta} - 3\underline{v}\delta\sqrt{\underline{v}+\delta} + \delta^2\left(\sqrt{\overline{v}-\delta} + \sqrt{\underline{v}+\delta}\right)\right)}{15\left(\overline{v}-\underline{v}\right)\delta}. \end{aligned}$$

The partial derivative of this expression with respect to $\delta$ yields

$$\frac{\partial x_{C2}^{IDV}}{\partial \delta} = \frac{8\overline{v}^2\sqrt{\overline{v}-\delta} - 8\overline{v}^{\frac{5}{2}} + 4\overline{v}\delta\sqrt{\overline{v}-\delta} - 4\underline{v}\delta\sqrt{\underline{v}+\delta}}{15\left(\overline{v}-\underline{v}\right)\delta} + \frac{8\underline{v}^2\sqrt{\underline{v}+\delta} - 8\underline{v}^{\frac{5}{2}} + 3\delta^2\left(\sqrt{\overline{v}-\delta} + \sqrt{\underline{v}+\delta}\right)}{15\left(\overline{v}-\underline{v}\right)\delta}$$

This partial derivative is negative, so equilibrium effort levels increase as the quality of the signal decreases. Further, $x_{C2}^{IDV}$ converges to $x_{i2}^{IIV}$ as $\delta$ increases. Since $n=2$, $x_{C2}^{IDV} = E\left(x_{I2}^{IDV}\left(V_1\right)\right)$; aggregate effort expenditures converge to $R_2^{IIV}$. This is consistent with the result that the presence of an information asymmetry decreases effort in the second period. As the value of this signal decreases, equilibrium effort levels get closer and closer to $x_{i2}^{IIV}$.

Next, consider the problem faced by the contest designer. Suppose that this contest designer can choose between two types of information revelation policies. First, she can publicly announce the value of the prize in contest, either before or after contestants have chosen their effort levels. Notice that, ex ante, both of these policies will result in expected equilibrium effort expenditures as in the IIV case. Second, she can privately reveal this value to the contestant who obtained it (the IDV case). If the contest designer seeks to minimize effort expenditures, then Proposition 8 implies that she will adopt a policy of publicly revealing the value of the prize before or after the contestants choose their effort levels. Adopting such a policy ensures that, ex ante, effort expenditures are expected to correspond to the IIV case. If the contest designer seeks to maximize effort expenditures she will choose to adopt a policy of privately revealing the value of the prize to the contestant who obtains the prize. Interestingly, this is the opposite of the predictions in a one-shot asymmetric information contest. As such, taking account of the incentives to acquire information is important when considering optimal information revelation policy. In rent seeking applications, this result offers support for the view that there is social benefit to public disclosure of information.

# 3 Conclusion

When contestants compete in a series of imperfectly discriminating contests, obtaining a prize in an early period may provide information regarding the value of the prize in later periods. I examine the case where the values of the prizes are positively related in a twice repeated imperfectly discriminating contest. If the incumbent privately observes the value of the prize in the first contest, then she is better informed than the challengers in the subsequent contest.

I find that in the second contest, the incumbent has a strictly lower ex ante probability of obtaining the prize than a challenger, despite expending (weakly) more effort than a challenger in expectation. The incumbent expends low effort for low values of the prize and high effort for high values of the prize; the incumbent's low probability of obtaining the prize when its value is low is such that the ex ante probability of obtaining the prize is lower than that of a challenger.

Since the incumbent expends low effort for low values of the prize, the challengers face an analogue of the winner's curse, and reduce their second period effort expenditures relative to the symmetric information case as a result. This is sufficient to reduce aggregate effort expenditure in the second contest relative to the IIV case, despite the fact that the incumbent's expected effort expenditures may have increased relative to the IIV case.

The incumbent's ex ante expected utility is strictly higher than in the IIV case; the incumbent obtains an information rent. This information rent creates an increased incentive to obtain the prize in the first contest, which increases aggregate effort expenditures in the first contest. This incentive is sufficiently high to increase total effort expenditure over both contests, offsetting the decrease in expected effort expenditure in $t = 2$ caused by the information asymmetry.

# Appendix A

**Proposition 1** *There is a unique Nash equilibrium in $t = 2$ of the IDV case.*

**Proof.** Define the function

$$
\begin{aligned}
g(x) \equiv\; & \frac{(n-2)}{x(n-1)} \int_{\underline{v}}^{q(x(n-1))} \int_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1 \\
& + \sqrt{\frac{(n-1)}{x}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} f(v_1, v_2) dv_2 dv_1 \\
& + F_V(q(x(n-1))) - n.
\end{aligned}
$$

Notice that $g(x) = 0$ satisfies (3), which defines an equilibrium. Note that $q(\frac{\underline{v}}{n-1}) = \underline{v}$.

$$
\begin{aligned}
g\left(\frac{\underline{v}}{n-1}\right) &= \sqrt{\frac{(n-1)}{\left(\frac{\underline{v}}{n-1}\right)}} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} dF_V(v_1) - n \\
&= \frac{(n-1)}{\sqrt{\underline{v}}} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} dF_V(v_1) - n
\end{aligned}
$$

Now, suppose that $g(\frac{\underline{v}}{n-1}) \leq 0$. In this case, $\left(\frac{(n-1)\left(E\left(\sqrt{E(V_2|V_1)}\right)\right)}{n}\right)^2 \leq \underline{v} \leq E(V_2 \mid \underline{v})$. That is, there is interior equilibrium. If $g(\frac{\underline{v}}{n-1}) > 0$, there need not be an interior equilibrium. However,

$$\lim_{x \to \infty} g(x) = 1 - n < 0.$$

Thus, either there is an interior equilibrium, or the intermediate value theorem assures at least one finite value of $x$ where $g(x) = 0$. If there is an interior equilibrium, then it has a unique closed form solution. To prove the uniqueness of a non-interior equilibrium note that:

$$\frac{\partial g(x)}{\partial x} = \frac{(n-2)(n-1)\int_{\underline{v}}^{\infty} v_2 f(q(x(n-1)), v_2) dv_2 q'(x(n-1))}{x(n-1)}$$
$$-f_V(q(x(n-1)))q'(x(n-1))(n-1)^2$$
$$+f_V(q(x(n-1)))q'(x(n-1))(n-1)$$
$$-\frac{\sqrt{(n-1)}\int_{q(x(n-1))}^{\infty}\int_{\underline{v}}^{\infty}\sqrt{E(V_2 \mid v_1)} f(v_1, v_2) dv_2 dv_1}{2x^{\frac{3}{2}}}$$
$$-\frac{(n-2)\int_{\underline{v}}^{q(x(n-1))}\int_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1}{x^2(n-1)}.$$

If there is not an interior equilibrium, $E(V_2 \mid q(x(n-1))) = x(n-1)$. Using this to reduce the above expression yields:

$$\frac{\partial g(x)}{\partial x} = -\frac{\sqrt{(n-1)}\int_{q(x(n-1))}^{\infty}\int_{\underline{v}}^{\infty}\sqrt{E(V_2 \mid v_1)} f(v_1, v_2) dv_2 dv_1}{2x^{\frac{3}{2}}}$$
$$-\frac{(n-2)\int_{\underline{v}}^{q(x(n-1))}\int_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1}{x^2(n-1)}.$$

Since this expression is negative, $g(x)$ is monotonically decreasing in $x$, which means that the equilibrium whose existence was shown above is unique. ■

**Proposition 2** *In the IDV case, the ex ante expected effort expenditure of the incumbent is weakly greater than that of a challenger. If $n = 2$, or there is an interior equilibrium, the incumbent's ex ante expected effort level is equal to that of a challenger, otherwise the inequality is strict.*

**Proof.** Rearranging (3), which characterizes equilibrium effort yields:

$$\begin{aligned} x_{C2}^{IDV} - E\left(x_{I2}^{IDV}(v_1)\right) &= \left(\frac{n-2}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 \le q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ &\quad - x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-2) \end{aligned}$$

Note that the right hand side of this equation is equal to zero if $n = 2$, or if the equilibrium in interior ($q\left(x_{C2}^{IDV}(n-1)\right) = \underline{v}$), yielding the desired result. Now suppose the equilibrium is not interior, $n > 2$, and that $x_{C2}^{IDV} \ge E\left(x_{I2}^{IDV}(v_1)\right)$. This implies that:

$$\left(\frac{n-2}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 \le q\left(x_{C2}^{IDV}(n-1)\right)}\right) \ge x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-2).$$

This simplifies to

$$\begin{aligned} E\left(V_2 \mid V_1 \le q\left(x_{C2}^{IDV}(n-1)\right)\right) &\ge x_{C2}^{IDV}(n-1) \\ &= E\left(V_2 \mid q\left(x_{C2}^{IDV}(n-1)\right)\right). \end{aligned}$$

Since $E(V_2 \mid v_1)$ is strictly increasing in $v_1$, this is a contradiction. ■

**Proposition 3** *When $V_2$ is positively regression dependent on $V_1$, the incumbents ex ante expected probability of obtaining the prize is strictly less than that of a challenger.*

**Proof.** First consider the case where $n = 2$, or there is an interior equilibrium. Recall that, when $n = 2$, or there is an interior equilibrium, $x_{C2}^{IDV} = E\left(x_{I2}^{IDV}(v_1)\right)$. In this case, note that the probability contestant $j \in \mathbf{C}$ will obtain the prize, $p_{j2}(x_{I2}(v_1), \mathbf{x_C}) = \frac{x_{j2}}{\left(x_{i2}(v_1) + x_{j2} + \sum_{k \in \mathbf{C}_{-j}} x_{k2}\right)}$, is strictly convex in $x_{I2}(v_1)$. Jensen's Inequality yields:

$$E\left(p_{j2}(x_{I2}(v_1), \mathbf{x_C})\right) > p_{j2}\left(E x_{I2}^{IDV}(v_1), \mathbf{x_C}\right).$$

Further, since $p_{I2}(x_{i2}(v_1), \mathbf{x_C}) = \frac{x_{I2}(v_1)}{\left(x_{I2}(v_1) + \sum_{k \in \mathbf{C}} x_{k2}\right)}$ is strictly concave in $x_{I2}(v_1)$, Jensen's Inequality also tells us that:

$$E\left(p_{I2}\left(x_{I2}^{IDV}(v_1), \mathbf{x_C}\right)\right) < p_{I2}\left(E\left(x_{I2}^{IDV}(v_1)\right), \mathbf{x_C}\right).$$

Dividing both sides of $x_{C2}^{IDV} = E\left(x_{I2}^{IDV}(v_1)\right)$ by $E\left(x_{I2}^{IDV}(v_1)\right) + (n-1)\,x_{C2}^{IDV}$, and using the above inequalities yields:

$$\begin{aligned} E\left(p_{j2}\left(x_{I2}\left(v_1\right), \mathbf{x_C}\right)\right) &> p_{j2}\left(E\left(x_{I2}^{IDV}\left(v_1\right)\right), \mathbf{x_C}\right) \\ &= p_{I2}\left(E\left(x_{I2}^{IDV}\left(v_1\right)\right), \mathbf{x_C}\right) \\ &> E\left(p_{I2}\left(x_{I2}\left(v_1\right), \mathbf{x_C}\right)\right). \end{aligned}$$

When $n > 2$ and there is not an interior equilibrium $x_{C2}^{IDV} \leq E\left(x_{I2}^{IDV}(v_1)\right)$. The ex ante probability that the incumbent obtains the good is given by

$$\begin{aligned} & E\left(p_{I2}\left(x_{I2}\left(v_1\right), \mathbf{x_C}\right)\right) \\ = & \left(1 - F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\right) \\ & -\sqrt{x_{C2}^{IDV}\left(n-1\right)}E\left(\frac{1}{\sqrt{E\left(V_2 \mid V_1\right)}}\mathbf{1}_{V_1 \geq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

The ex ante probability that a challenger $j \in \mathbf{C}$ obtains the good is given by

$$\begin{aligned} & E\left(p_{j2}\left(x_{I2}\left(v_1\right), \mathbf{x_C}\right)\right) \\ = & \frac{F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)}{(n-1)} \\ & +\sqrt{\frac{x_{C2}^{IDV}}{(n-1)}}E\left(\frac{1}{\sqrt{E\left(V_2 \mid V_1\right)}}\mathbf{1}_{V_1 \geq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

Suppose that $E\left(p_{j2}\left(x_{I2}\left(v_1\right), \mathbf{x_C}\right)\right) < E\left(p_{I2}\left(x_{I2}\left(v_1\right), \mathbf{x_C}\right)\right)$. This simplifies to

$$\begin{aligned} & 1 - \frac{1}{n\left(\left(1 - F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\right)\right)} \\ > & \sqrt{x_{C2}^{IDV}\left(n-1\right)}E\left(\frac{1}{\sqrt{E\left(V_2 \mid V_1\right)}}|V_1 \geq q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \\ > & 1. \end{aligned}$$

This is a contradiction. ■

**Proposition 4** *The equilibrium effort expenditure of a contestant in* $t = 2$ *of the IIV case is strictly greater than the equilibrium effort expenditure of a challenger in*

*$t = 2$ of the IDV case if and only if*

$$\begin{aligned} & \frac{(n-2)}{(n-1)(n-F_V(q(B)))} E\left(V_2 1_{V_1 \leq q(B)}\right) \\ & + \frac{(n-1)\sqrt{E(V)}}{n(n-F_V(q(B)))} E\left(\sqrt{V_2} 1_{V_1 \geq q(B)}\right) \\ < \quad & \frac{E(V)(n-1)}{n^2}, \end{aligned}$$

*where $B \equiv \frac{E(V)(n-1)^2}{n^2} = x_{i2}^{IIV}(n-1)$.*

**Proof.** Notice that, when $n = 2$, Jensen's inequality implies that (4) holds. Further, notice that (4) states that $g\left(\frac{E(V)(n-1)}{n^2}\right) < 0$ ($g(x)$ was defined in the proof of Proposition 1). Recall that in the proof of Proposition 1 it was shown that $g(x)$ is a monotonically decreasing function, and that $g(x) = 0$ defines the unique equilibrium of the game. So if $x_{C2}^{IDV} < x_{i2}^{IIV} = \frac{E(V)(n-1)}{n^2}$, then $g\left(x_{C2}^{IDV}\right) > g\left(x_{i2}^{IIV}\right) = g\left(\frac{E(V)(n-1)}{n^2}\right)$. Since $g\left(x_{C2}^{IDV}\right) = 0$ in equilibrium, $g\left(x_{i2}^{IIV}\right) = g\left(\frac{E(V)(n-1)}{n^2}\right) < 0$, which is the condition given in (4). To see that (4) implies $x_{C2}^{IDV} < x_{i2}^{IIV}$, consider $g\left(x_{i2}^{IIV}\right) = g\left(\frac{E(V)(n-1)}{n^2}\right) < 0$. Since $g\left(x_{C2}^{IDV}\right) = 0$, and $g(x)$ is monotonically decreasing in $x$, it must be the case that $x_{C2}^{IDV} < x_{i2}^{IIV}$. ■

**Proposition 5** *If (4) is satisfied, then the ex ante expected utility of a challenger is strictly less in the IDV case than the IIV case.*

**Proof.** Define the following function:

$$h(x) \equiv \frac{1}{(n-1)^2} E\left(V_2 \mathbf{1}_{V_1 \leq q(x(n-1))}\right) + \frac{x\left(1 - F_V(q(x(n-1)))\right)}{(n-1)}.$$

Note that:

$$\begin{aligned} h'(x) \quad = \quad & \frac{1}{(n-1)} \int_{\underline{v}}^{\infty} v_2 f\left(q(x(n-1)), v_2\right) q'(x(n-1))\, dv_2 \\ & + \frac{\left(1 - F_V(q(x(n-1)))\right)}{(n-1)} - \int_{\underline{v}}^{\infty} x f\left(q(x(n-1)), v_2\right) q'(x(n-1))\, dv_2 \end{aligned}$$

But if $x(n-1) > \underline{v}$, then $x(n-1) = E(V_2 \mid q(x(n-1)))$. Plugging this in simplifies this expression down to the following:

$$h'(x) \geq \frac{(1 - F_V(q(x(n-1))))}{(n-1)} > 0$$

Since $h'(x) > 0$, and (4) is satisfied my assumption, $x_{C2}^{IDV} < \frac{E(V)(n-1)}{n^2} = x_{i2}^{IIV}$. Thus, $h\left(x_{C2}^{IDV}\right) < h(\frac{E(V)(n-1)}{n^2})$. Note that (where the second line follows from the definition of conditional probability):

$$\begin{aligned}
h\left(\frac{E(V)(n-1)}{n^2}\right) &= \frac{1}{(n-1)^2}\int_{\underline{v}}^{\infty}\int_{\underline{v}}^{q(B)} v_2 f(v_1,v_2)dv_1dv_2 + \frac{E(V)}{n^2}(1 - F_V(q(B))) \\
&= \frac{F_V(q(B))}{(n-1)^2}E(V_2 \mid V_1 \leq q(B)) + \frac{E(V_2)}{n^2}(1 - F_V(q(B))) \\
&\leq \frac{F_V(q(B))}{(n-1)^2}\frac{E(V)(n-1)^2}{n^2} + \frac{E(V_2)}{n^2}(1 - F_V(q(B))) \\
&= \frac{E(V)}{n^2}F_V(q(B)) + \frac{E(V_2)}{n^2}(1 - F_V(q(B))) \\
&= \frac{E(V)}{n^2}.
\end{aligned}$$

■

**Proposition 6** *If (4) is satisfied the ex ante expected utility of the incumbent is strictly greater in the IPV case than in the IIV case.*

**Proof.** Notice that $E\left(U_{i2}^{IIV}\right) < E\left(U_{I2}^{IDV}\right)$ when

$$\begin{aligned}
&E(V) + \left(\frac{n-3}{n-1}\right)E\left(V_2\mathbf{1}_{V_1<q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\
&-x_{C2}^{IDV}(n+1) - xF_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-3) > \frac{E(V)}{n^2}.
\end{aligned}$$

This expression can be rewritten as

$$\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)(n-1) > R_2^{IDV} - R_2^{IIV}.$$

Similarly, $E\left(U_{i2}^{IIV}\right) > E\left(U_{I2}^{IDV}\right)$ when

$$\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)(n-1) < R_2^{IDV} - R_2^{IIV}.$$

Likewise, $E\left(U_{i2}^{IIV}\right) = E\left(U_{I2}^{IDV}\right)$ when

$$\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)(n-1) = R_2^{IDV} - R_2^{IIV}.$$

Define the function

$$r(x) \equiv \sqrt{x(n-1)}E\left(\sqrt{V_2 \mathbf{1}_{V_1 \geq q(x(n-1))}}\right) + x(n-1)F_V(q(x(n-1))),$$

which corresponds to $R_2^{IDV}$, and

$$w(x) = \left(\frac{1}{n-1}\right)E\left(V_2 \mathbf{1}_{V_1 < q(x(n-1))}\right) + x(1 - F_V(q(x(n-1)))),$$

which corresponds to $E\left(U_{C2}^{IDV}\right)(n-1)$. Note that

$$r'(x) = (n-1)F_V(q(x(n-1))) + \frac{1}{2}\sqrt{\frac{n-1}{x}}E\left(\sqrt{V_2 \mathbf{1}_{V_1 \geq q(x(n-1))}}\right),$$

and that

$$w'(x) = 1 - F_V(q(x(n-1))).$$

Now notice that $r'(x) > w'(x) > 0$. Since $r(x)$ and $w(x)$ are both strictly monotonically increasing, and $r'(x) > w'(x)$, the expressions $E\left(U_{i2}^{IIV}\right)(n-1) - w(x)$ and $r(x) - R_2^{IIV}$ intersect only once. Let $\widetilde{x} \equiv \left\{x : r(x) - R_2^{IIV} = E\left(U_{i2}^{IIV}\right)(n-1) - w(x)\right\}$, which has a single element. Notice that if $x_{C2}^{IDV} = \widetilde{x}$, then the IDV incumbent's expected utility in the IDV case is the same as in the IIV case. It has been proven that $E\left(U_{C2}^{IDV}\right) < E\left(U_{i2}^{IIV}\right)$, which implies that $x_{C2}^{IDV} < \widetilde{x}$. Thus, $E\left(U_{i2}^{IIV}\right)(n-1) - w\left(x_{C2}^{IDV}\right) > E\left(U_{i2}^{IIV}\right)(n-1) - w(\widetilde{x})$. Also, $x_{C2}^{IDV} < \widetilde{x}$ implies that $r\left(x_{C2}^{IDV}\right) - R_2^{IIV} < r(\widetilde{x}) - R_2^{IIV}$. Since $r'(x) > w'(x) > 0$,

$$\begin{aligned} & E\left(U_{i2}^{IIV}\right)(n-1) - w\left(x_{C2}^{IDV}\right) - E\left(U_{i2}^{IIV}\right)(n-1) - w(\widetilde{x}) \\ < \; & r(\widetilde{x}) - R_2^{IIV} - \left(r\left(x_{C2}^{IDV}\right) - R_2^{IIV}\right). \end{aligned}$$

This simplifies to

$$\begin{aligned} r\left(x_{C2}^{IDV}\right) + w\left(x_{C2}^{IDV}\right) &< w(\widetilde{x}) + r(\widetilde{x}) \\ &= E\left(U_{i2}^{IIV}\right)(n-1) + R_2^{IIV}. \end{aligned}$$

That is, $\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)(n-1) > R_2^{IDV} - R_2^{IIV}$. ■

**Proposition 7** *If (4) is satisfied, ex ante expected effort expenditures are strictly lower in the IDV case than in the IIV case.*

**Proof.** Suppose that $R_2^{IDV} > R_2^{IIV}$. Since $E\left(U_{I2}^{IDV}\right) > E\left(U_{i2}^{IIV}\right)$

$$R_2^{IIV} - R_2^{IDV} > \left(E\left(U_{C2}^{IDV}\right) - E\left(U_{i2}^{IIV}\right)\right)(n-1).$$

But this can be rewritten as

$$\begin{aligned} & \left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right) n(n-1) + 2x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) \\ & -2\left(\frac{1}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ > \; & \left(E\left(U_{C2}^{IDV}\right) - E\left(U_{i2}^{IIV}\right)\right)(n-1). \end{aligned}$$

Since, $x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) \geq \left(\frac{1}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right)$, and $E\left(U_{C2}^{IDV}\right) < E\left(U_{i2}^{IIV}\right)$, the LHS of this inequality is positive. $R_2^{IDV} > R_2^{IIV}$ implies that the LHS is negative, a contradiction. ■

**Proposition 8** *When the equilibrium in $t = 2$ of the IDV case is not interior , total effort expenditures in the IPV case, $R^{IDV}$, strictly exceed those of the IIV case. If the equilibrium in $t = 2$ of the IDV case is interior then $R^{IDV} = R^{IIV}$.*

**Proof.** In equilibrium, the difference between the IDV incumbent's ex ante expected utility and that of the challenger is:

$$\begin{aligned} E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right) &= E(V) + \left(\frac{n-3}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ & -\frac{1}{(n-1)^2} E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ & -\frac{x_{C2}^{IDV}\left(1 - F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)\right)}{(n-1)}. \end{aligned}$$

Notice that total effort expenditure in the IPV case will increase relative to the IIV case if:

$$\begin{aligned} \frac{2E(V)(n-1)}{n} \leq & \frac{\left(E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right)(n-1)}{n} + \\ & n x_{C2}^{IDV} + x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-2) \\ & -\left(\frac{n-2}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

This condition simplifies to:

$$E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right)\leq F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)x_{C2}^{IDV}(n-1).$$

Since $E(V_2 \mid v_1)$ is strictly increasing in $v_1$ the inequality is strict if the equilibrium in $t=2$ is not interior. If the equilibrium is interior, then $R^{IDV}=R^{IIV}$. ■

**Proposition 9** *The equilibrium expenditure in the IDV case of a challenger and of the incumbent is decreasing in n. The ex ante expected aggregate effort expenditures in $t=2$ is increasing in n.*

**Proof.** Recall that $g(x)=0$ satisfies (3), which defines an equilibrium

$$\begin{aligned} g(x) &= \frac{(n-2)}{x(n-1)}\int_{\underline{v}}^{q(x(n-1))}\int_{\underline{v}}^{\infty}v_2 f(v_1,v_2)dv_2dv_1 \\ &+\sqrt{\frac{(n-1)}{x}}\int_{q(x(n-1))}^{\infty}\int_{\underline{v}}^{\infty}\sqrt{E(V_2 \mid v_1)}f(v_1,v_2)dv_2dv_1 \\ &+F_V(q(x(n-1)))-n. \end{aligned}$$

The partial derivative with respect to $x$ is

$$\begin{aligned} \frac{\partial g}{\partial x} &= -\frac{\sqrt{(n-1)}\int_{q(x(n-1))}^{\infty}\int_{\underline{v}}^{\infty}\sqrt{E(V_2 \mid v_1)}f(v_1,v_2)dv_2dv_1}{2x^{\frac{3}{2}}} \\ &-\frac{(n-2)\int_{\underline{v}}^{q(x(n-1))}\int_{\underline{v}}^{\infty}v_2 f(v_1,v_2)dv_2dv_1}{x^2(n-1)}<0. \end{aligned}$$

The partial derivative with respect to $n$ is

$$\begin{aligned}\frac{\partial g}{\partial n} &= \frac{1}{x\left(n-1^2\right)}\int\limits_{\underline{v}}^{q(x(n-1))}\int\limits_{\underline{v}}^{\infty} v_2 f(v_1,v_2)dv_2dv_1 \\ &\quad + \frac{1}{2\sqrt{x\left(n-1\right)}}\int\limits_{q(x(n-1))}^{\infty}\int\limits_{\underline{v}}^{\infty}\sqrt{E\left(V_2 \mid v_1\right)}f(v_1,v_2)dv_2dv_1 - 1.\end{aligned}$$

(3) immediately demonstrates that this expression is negative. Since both of these partial derivatives are negative,

$$\frac{dx}{dn} = -\frac{\left(\frac{\partial g}{\partial n}\right)}{\left(\frac{\partial g}{\partial x}\right)} < 0.$$

That is $\frac{dx_{C2}^{IDV}}{dn} < 0$.

Next, note that

$$\begin{aligned}\frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial n} &= x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) - x \\ &\quad + \frac{1}{2}\sqrt{\frac{x_{C2}^{IDV}}{n-1}}\int\limits_{q(x(n-1))}^{\infty}\int\limits_{\underline{v}}^{\infty}\sqrt{E\left(V_2 \mid v_1\right)}f(v_1,v_2)dv_2dv_1.\end{aligned}$$

$$\begin{aligned}\frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial x_{C2}^{IDV}} &= \left(n-1\right) F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) - n \\ &\quad + \frac{1}{2}\sqrt{\frac{n-1}{x_{C2}^{IDV}}}\int\limits_{q(x(n-1))}^{\infty}\int\limits_{\underline{v}}^{\infty}\sqrt{E\left(V_2 \mid v_1\right)}f(v_1,v_2)dv_2dv_1.\end{aligned}$$

Utilizing (3), it is straightforward to show that both of these are positive. Plugging these partial derivatives into

$$\frac{dE\left(x_{I2}^{IDV}\right)}{dn} = \frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial n} + \frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial x_{C2}^{IDV}}\frac{dx_{C2}^{IDV}}{dn},$$

and simplifying demonstrates that $\frac{dE\left(x_{I2}^{IDV}\right)}{dn} < 0$. Next, note that

$$\frac{\partial R_2^{IDV}}{\partial n} = x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) + \frac{1}{2}\sqrt{\frac{x_{C2}^{IDV}}{n-1}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1.$$

$$\frac{\partial R_2^{IDV}}{\partial x_{C2}^{IDV}} = (n-1) F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) + \frac{1}{2}\sqrt{\frac{n-1}{x_{C2}^{IDV}}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1.$$

These partial derivatives are positive. Plugging them into

$$\frac{dR_2^{IDV}}{dn} = \frac{\partial R_2^{IDV}}{\partial n} + \frac{\partial R_2^{IDV}}{\partial x_{C2}^{IDV}} \frac{dx_{C2}^{IDV}}{dn},$$

and simplifying demonstrates that $\frac{dR_2^{IDV}}{dn} > 0$. ■

# Appendix B

This appendix contains two alternative ways of modeling an incumbency advantage. Both of these maintain the information structure of the IIV case, such that information complete in $t = 1, 2$.

## Status Quo Bias (SQB)

One way in which an incumbent might have an advantage over a challenger is through an increased probability of winning the subsequent contest for any vector of effort $\mathbf{x}_2$. That is, by virtue of holding the high ground, the incumbent has an exogenously higher probability of winning than she would otherwise have. I call such an incumbency advantage a status quo bias.

Consider the case in which $v_1$ and $v_2$ are independent draws from the distribution $F_V$ (the information structure found in the IIV case). To model a status quo bias, the contest success function is modified such that the probability that contestant $i$ obtains the prize in $t = 2$ is now given by

$$\widetilde{p}_{i2}\left(x_{i2}, \mathbf{x}_{-i2}\right) = \frac{x_{i2} + \theta \mathbf{1}_{\{i=I\}}}{x_{i2} + \theta + \sum\limits_{j \in \mathbf{N}_{-i}} x_{j2}},$$

where $\theta > 0$ is added to the aggregate effort expenditures in $t = 2$, and the probability $\theta / \left(\theta + \sum_{i \in \mathbf{N}} x_{i2}\right) > 0$ represents the status quo bias. This is similar to the incumbent having a negative fixed cost of effort. However, it differs in that the incumbent is not awarded $\theta$ if she were to expend zero effort. Notice that $\theta / \left(\theta + \sum_{i \in \mathbf{N}} x_{i2}\right)$ is decreasing in $\sum_{i \in \mathbf{N}} x_{i2}$. This captures the idea that an incumbent has an increased probability of obtaining the prize in $t = 2$, but that challengers are at less of a disadvantage as they increase their effort. If $\sum_{i \in \mathbf{N}} x_{i2} = 0$, then the incumbent wins with certainty. As such there is no need to separately define the border case in which no contestant expends any effort. In $t = 1$, the contest success function is unchanged from that of the IIV and IDV cases.

I now turn attention to the incumbent's problem in the $t = 2$. (as before, player $I$ is the incumbent). The incumbent's expected utility is

$$U_{I2}^{SQB} \equiv \int\limits_{\underline{v}}^{\infty} \widetilde{p}_{i2}\left(x_{I2}, \mathbf{x}_C\right) v_2 dF_V(v_2) - x_{I2}.$$

The partial derivative is given by

$$\frac{E(V) \sum\limits_{j \in \mathbf{C}} x_{j2}}{\left(x_{I2} + \theta + \sum\limits_{j \in \mathbf{C}} x_{j2}\right)^2} - 1.$$

Similarly, the expected utility of contestant $j \in \mathbf{C}$ is

$$U_{j2}^{SQB} \equiv \int\limits_{\underline{v}}^{\infty} \widetilde{p}_{j2}\left(x_{j2}, \mathbf{x}_{-j2}\right) v_2 dF_V(v_2) - x_{j2}$$

with partial derivative

$$\frac{E\left(V\right)\left(\sum_{k\in\mathbf{C}_{-j}} x_{k2}+x_{I2}+\theta\right)}{\left(x_{I2}+\theta+\sum_{j\in\mathbf{C}} x_{j2}\right)^{2}}-1.$$

Reasoning identical to that used in the IDV case demonstrates that the challengers will exert the same amount of effort in equilibrium. In the SQB case, I denote equilibrium effort by the incumbent as $x_{I2}^{SQB}$ and equilibrium effort of a challenger as $x_{C2}^{SQB}$. The magnitude of $\theta$ determines whether contestants will expend positive effort in equilibrium.

First, consider $\theta \geq E\left(V\right)$. Notice that when $\theta \geq E\left(V\right)$, a challenger's will optimally expend zero effort. Also, when $\sum_{j\in\mathbf{C}} x_{j2}=0$, then the incumbent's best response is to expend zero effort because she will obtain the prize with certainty regardless of expenditure. Thus, when $\theta > E\left(V\right)$, $x_{I2}^{SQB}=x_{C2}^{SQB}=0$. The intuition of this scenario is clear: when the incumbent has an advantage so significant that $x_{C2}^{SQB} \geq E\left(V\right)$ just to have an equal probability of winning the prize (even when the incumbent doesn't expend any effort), the challengers will not expend any effort. In this case, the incumbent obtains the prize with certainty. Thus, if $\theta \geq E\left(V\right)$, the ex ante value of obtaining the good in $t=1$ is $2E\left(V\right)$.

Now consider $\theta \in \left[E\left(V\right)\left(n-1\right)/n^{2}, E\left(V\right)\right)$. The status quo bias is significant enough that $x_{I2}^{SQB}=0$. The first order condition of a challenger holds, and

$$x_{C2}^{SQB}=\frac{\left(n-2\right)E\left(V\right)-2\left(n-1\right)\theta+\sqrt{\left(n-2\right)^{2}E\left(V\right)^{2}+4E\left(V\right)\left(n-1\right)\theta}}{2\left(n-1\right)^{2}}.$$

In this case, the status quo bias is not so large that a challenger will not attempt to obtain the prize, but it is large enough that that the incumbent does not expend any effort. Notice that this is the case if $\theta \geq x_{i2}^{IIV}$.

Next, consider $\theta \in \left(0, E\left(V\right)\left(n-1\right)/n^{2}\right)$. Here every contestant's first order condition holds. Solving the set of $n$ simultaneous equations yields equilibrium effort levels $x_{C2}^{SQB} = x_{i2}^{IIV}$, and $x_{I2}^{SQB} = x_{i2}^{IIV} - \theta$. Notice that $x_{I2}^{SQB} > 0$ only when $\theta < E(V)(n-1)/n^2$.

So, to summarize, the equilibrium effort levels of a challenger in $t = 2$ of the SQB case are given by

$$x_{C2}^{SQB} = \begin{cases} \frac{E(V_2)(n-1)}{n^2} & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{(n-2)E(V)-2(n-1)\theta+\sqrt{(n-2)^2E(V)^2+4E(V)(n-1)\theta}}{2(n-1)^2} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Notice that $x_{C2}^{SQB}$ is decreasing in $n$ when $\theta < E(V)$, and that $\lim_{n\to\infty} x_{C2}^{SQB} = 0$ as in the IIV case. The equilibrium effort expenditure of the incumbent is

$$x_{I2}^{SQB} = \begin{cases} \frac{E(V)(n-1)}{n^2} - \theta & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ 0 & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Which is decreasing in $\theta$ and $n$ when $\theta < E(V)(n-1)/n^2$. Since

$$\lim_{n\to\infty} E(V)(n-1)/n^2 = 0$$

for any $\theta > 0$ there exists some $n$ large enough that $\theta > E(V)(n-1)/n^2$ and $x_{I2}^{SQB} = 0$ above this $n$. Therefore $\lim_{n\to\infty} x_{I2}^{SQB} = 0$.

The equilibrium aggregate effort expenditures in $t = 2$ of the SQB case, $R_2^{SQB}$, is given by

$$R_2^{SQB} = \begin{cases} \frac{E(V)(n-1)}{n} - \theta & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{(n-2)E(V)-2(n-1)\theta+\sqrt{(n-2)^2E(V)^2+4E(V)(n-1)\theta}}{2(n-1)} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Notice that $R_2^{SQB}$ is decreasing in $\theta$ and $n$ when $\theta < E(V)$. As such

$$\begin{aligned} \lim_{n\to\infty} R_2^{SQB} &= \lim_{n\to\infty} \frac{(n-2)E(V) - 2(n-1)\theta + \sqrt{(n-2)^2 E(V)^2 + 4E(V)(n-1)\theta}}{2(n-1)} \\ &= E(V) - \theta. \end{aligned}$$

The equilibrium expected utility of the incumbent in the SQB case is given by

$$U_{I2}^{SQB}=\begin{cases}\frac{E(V)}{n^2}+\theta & \text{if} \quad \theta\in\left(0,\frac{E(V)(n-1)}{n^2}\right)\\ \frac{\sqrt{(n-2)^2E(V)^2+4E(V)(n-1)\theta}-(n-2)E(V)}{2} & \text{if} \quad \theta\in\left[\frac{E(V)(n-1)}{n^2},E(V)\right)\\ E(V) & \text{if} \quad \theta\in[E(V),\infty).\end{cases}$$

When $\theta\in(0,E(V)(n-1)/n^2)$, the incumbent's expected utility has increased by exactly $\theta$ relative to the IIV case. For $\theta\in[E(V)(n-1)/n^2,E(V))$, the incumbent's expected utility is increasing at a decreasing rate in $\theta$. Once $\theta\geq E(V)$, the status quo bias is so large that the incumbent wins the prize with certainty without expending any effort. As such increasing the magnitude of $\theta$ does not increase her expected utility. Likewise increasing $n$ does not affect $U_{I2}^{SQB}$ when $\theta\geq E(V)$. When $\theta<E(V)$, $U_{I2}^{SQB}$ is decreasing in $n$. Because $\lim_{n\to\infty}E(V)(n-1)/n^2=0$, $\lim_{n\to\infty}U_{I2}^{SQB}=0$.

The equilibrium expected utility of a challenger in the SQB case is given by

$$U_{C2}^{SQB}=\begin{cases}\frac{E(V)}{n^2} & \text{if} \quad \theta\in\left(0,\frac{E(V)(n-1)}{n^2}\right)\\ \frac{E(V)(n(n-2)+2)+2\theta(n-1)-n\sqrt{(n-2)^2E(V)^2+4E(V)(n-1)\theta}}{2(n-1)^2} & \text{if} \quad \theta\in\left[\frac{E(V)(n-1)}{n^2},E(V)\right)\\ 0 & \text{if} \quad \theta\in[E(V),\infty).\end{cases}$$

Notice that when $\theta\in(0,E(V)(n-1)/n^2)$, $U_{C2}^{SQB}=U_{i2}^{IIV}$. For

$$\theta\in\left[E(V)(n-1)/n^2,E(V)\right)$$

the expected utility of a challenger is decreasing in $\theta$. Once $\theta\geq E(V)$, a challenger does not obtain the prize with certainty, and has an expected utility of zero as a result. Notice that when $\theta<E(V)$, $U_{C2}^{SQB}$ is decreasing in $n$ and that $\lim_{n\to\infty}U_{C2}^{SQB}=0$

An interesting result arises when $\theta\in(0,E(V)(n-1)/n^2]$. The expected utility of the incumbent has increased by $\theta$ relative to the IIV case, and the expected utility of a challenger remains unchanged relative to the benchmark case. Further, $R_2^{IIV}-R_2^{SQB}=\theta$. If a contest designer were concerned with the welfare of the contestants, and would also like to decrease total effort in $t=2$, choosing $\theta=E(V)(n-1)/n^2$ reduces equilibrium effort expenditures, and strictly increases the expected utility of the incumbent without reducing the expected utility of the challengers. Put another way, in a one-shot game, where effort is a social bad, choosing $\theta = E(V)(n-1)/n^2$ Pareto dominates $\theta < E(V)(n-1)/n^2$.

Turning attention to $t = 1$, note that the incentives the contestants face in $t = 1$ will be different, depending on the magnitude of $\theta$. Thus, each of the three cases outlined above must be considered individually. The expected utility of contestant $i$ is

$$\begin{aligned} U_{i1}^{SQB} &\equiv \int_{\underline{v}}^{\infty} p(x_{i1}, \mathbf{x}_{-i1}) v_1 dF_V(v_1) \\ &\quad -x_{i1} + \left(p(x_{i1}, \mathbf{x}_{-i1}) \left(U_{I2}^{SQB}\right)\right) \\ &\quad + (1 - p(x_{i1}, \mathbf{x}_{-i1})) \left(U_{C2}^{SQB}\right). \end{aligned}$$

The first period is, in essence, a contest in which the prize over which the contestants compete is $E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)$. The unique and symmetric equilibrium involves every contestant $i \in \mathbf{N}$ expending

$$x_{i1}^{SQB} \equiv \frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n^2}$$

in $t = 1$. The equilibrium aggregate effort expenditures in $t = 1$ is then

$$R_1^{SQB} \equiv n x_{i1}^{SQB} = \frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n}$$

and the equilibrium expected utility of contestant $i$ in $t = 1$ is

$$U_{i1}^{SQB} = \frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)}{n^2}.$$

Total equilibrium effort expenditures across both periods is given by

$$R^{SQB} \equiv \begin{cases} \frac{2E(V)(n-1)}{n} - \frac{\theta}{n} & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{\left(E(V)+\left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n} + (n-1)\, x_{C2}^{SQB} + x_{I2}^{SQB} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ \frac{E(V)(n-1)}{n} & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

When $\theta \in [E(V)(n-1)/n^2, E(V))$, I have not simplified $R^{SQB}$ due to space constraints. $R^{SQB} > R^{IIV}$ if

$$\frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n} + (n-1)\,x_{C2}^{SQB} + x_{I2}^{SQB} > \frac{2E(V)(n-1)}{n}.$$

When

$$\theta \in \left(0, E(V)(n-1)/n^2\right)$$

$R^{SQB} - R^{IIV} = -\theta/n$. When $\theta \in [E(V)(n-1)/n^2, E(V))$, $R^{SQB}$ is concave, and has a maximum value such that $R^{SQB} > R^{IIV}$. Once $\theta \in [E(V), \infty)$, $R^{SQB} = R^{IIV}$. Indeed, $R^{SQB} = R_1^{SQB}$.

A contest designer who seeks to maximize $R^{SQB}$, would would choose

$$\theta \in \left[E(V)(n-1)/n^2, E(V)\right).$$

Doing so ensures the the incumbent will not expend any effort. Effort expenditures in $t = 1$ more than make up for the decrease expenditures in $t = 2$. Further, if a contest designer sought to minimize effort expenditures (that is, maximize the sum of the contestants expected utility) she would choose $\theta = E(V)(n-1)/n^2$. Notice that this is the largest $\theta$ which does not reduce the expected utility of the challengers relative to the IIV case. Of interest is the fact that the optimal level of $\theta$ is positive, regardless of whether or not effort expenditures are a social bad.

# Cost Advantage (CST)

Another way to approach the concept of incumbency advantage is to allow the incumbent to have a cost advantage over the challenger. That is, allow the incumbent to have a lower marginal cost than the challenger. A model using this approach was introduced in Mehlum and Moene (2006). They model an infinitely repeated contest between two contestants in which a cost advantage is held by the contestant who obtained the prize in the previous period.

Below is a modified version of their model in which contestants compete in $t = 1$ with symmetric costs, and in $t = 2$, the incumbent has a lower marginal cost of effort than the challengers. Modeling it in this fashion allows me to examine the incentive to acquire this cost advantage when contestants are symmetric; and change in behavior in $t = 1$ relative to the IIV case is then attributable to the incumbents cost advantage. As such, the only difference between this model and the IIV case is that the incumbent has a cost of effort of $C_I(x_{I2}) = cx_{I2}$, where $c \in (0, 1)$.

In $t = 2$ the expected utility of the incumbent is

$$U_{I2}^{CST} \equiv \int_{\underline{v}}^{\infty} p_{I2}(x_{I2}, \mathbf{x}_C) v_2 dF_V(v_2) - cx_{I2}.$$

Similarly, the expected utility of contestant $j \in \mathbf{C}$ is

$$U_{j2}^{CST} \equiv \int_{\underline{v}}^{\infty} p_{j2}(x_{j2}, \mathbf{x}_{-i2}) v_2 dF_V(v_2) - x_{j2}.$$

This subgame has a unique subgame. I denote the equilibrium effort expenditure of the incumbent as $x_{I2}^{CS}$ and that of a challenger as $x_{C2}^{CST}$. The equilibrium effort levels are given by

$$\begin{aligned} x_{I2}^{CS} &= \frac{E(V)(n-1)(n(1-c)+2c-1)}{(n-1+c)^2} \\ x_{C2}^{CST} &= \frac{c(n-1)E(V)}{(n-1+c)^2}. \end{aligned}$$

The equilibrium aggregate effort expenditures in $t = 2$ is

$$R_2^{CST} \equiv \frac{E(V)(n-1)}{(n-1+c)}.$$

Further, the equilibrium expected utility of the incumbent is

$$U_{I2}^{CST} = \frac{cE(V)}{(n+c-1)^2}$$

and the equilibrium expected utility of a challenger is

$$U_{C2}^{CST} = \frac{c^2E(V)}{(n+c-1)^2}.$$

Next, consider contestant $i$'s expected utility in $t = 1$.

$$\begin{aligned} U_{i1}^{CST} \equiv & \int_{\underline{v}}^{\infty} p_{i1}\left(x_{i1}, \mathbf{x}_{-i1}\right) v_1 dF_V(v_1) - x_{i1} \\ & + p_{i1}\left(x_{i1}, \mathbf{x}_{-i1}\right)\left(\frac{c_I E\left(V_1\right)}{\left(n + c_I - 1\right)^2}\right) \\ & + \left(1 - p_{i1}\left(x_{i1}, \mathbf{x}_{-i1}\right)\right) \frac{c_I^2 E\left(V_1\right)}{\left(n + c_I - 1\right)^2}. \end{aligned}$$

Equilibrium effort expenditure in $t = 1$ is

$$x_{i1}^{CST} \equiv \frac{2\left(n-1\right) E\left(V\right)}{n^2\left(c+1\right)}.$$

Total equilibrium effort expenditures across $t = 1, 2$ is

$$R^{CST} \equiv \frac{2\left(n-1\right) E\left(V\right)}{n^2\left(c+1\right)} + \frac{E\left(V\right)\left(n-1\right)}{\left(n-1+c\right)}.$$

Notice that $R^{CST} > R^{IIV}$. This is because the reduced marginal cost causes the incumbent to increase her effort expenditures in $t = 2$ relative to the IIV case. In response, the challengers also increases their expenditures. Further, contestants in $t = 1$ increase their effort expenditures relative to the IIV case an attempt to obtain the incumbent cost advantage. Also, notice that $R^{CST}$ is monotonically decreasing in $c$; as the incumbents cost advantage increases, so does $R^{CST}$. This is in contrast to the status quo bias model discussed above. In that model, there were two competing effects, one of which increased effort, while the other decreased effort. As such, the effect of an incumbency advantage is sensitive to how it is modeled.

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